Indian Institute of Science Education and Research Thiruvananthapuram
PHY 312 Classical Mechanics, First Mid-Semester Examination, Batch 2009
13 September 2011, Duration: 60 minutes, Marks: 30

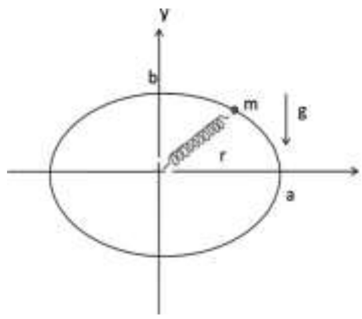
1. Show that the shortest distance between two points on the surface of a cylinder is a segment of an helix. (Hint: work in the cylindrical coordinate system and write the expression for a line segment $dl$. The integral of which must be minimised. Condition for a helical path: $dz/d\varphi$ = constant and $\rho$ = constant.) [10]

2. A sphere of radius r is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of radius R. Write down the constraint equation and derive the Lagrange's equations of motion by the method of undetermined multipliers. Find the frequency of small oscillations of the sphere about the bottom of the cylinder. [10]

3.. Consider a vertical plane in a uniform gravitational field. A particle moves on this plane under the influence of gravity and an additional force $F = -kr^{n-1}$, where r is the distance of the particle from an origin lying on the same plane and $n \neq 0$ or 1. Choose appropriate generalized coordinates and find Lagrange's equations of motion. Is the angular momentum about the origin conserved? Explain. [10]

Indian Institute of Science Education and Research Thiruvananthapuram
PHY 312 Classical Mechanics, Second Mid-Semester Examination, Batch 2009
17 October 2011, Duration: 60 minutes, Marks: 30

1. A point mass $m$ is constrained to move on an ellipse in a vertical plane, parameterized by $x= a\cos\theta$, $y=b\sin\theta$, where $a$, $b > 0$ and $a \neq b$. The particle is connected to the origin by a spring (of spring constant k), as shown in the diagram, and is subject to gravity.



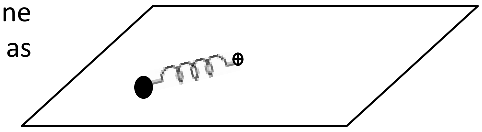

(i) Write the Langrangian and Lagrange's equation of motion.
(ii) Find any equilibrium points along the particle's motion, giving the conditions under which they exist.
(iii) Determine the stabilities of the two equilibrium points at $\theta = \pm\pi/2$.

2. A particle, moving under an attractive central force, has a circular orbit with the centre of force lying on the circumference of the circle. Show that the force varies as $1/r^5$ and calculate the period of the orbit, given $E$ and $l$ as the constants of motion.

3. A particle is thrown up vertically with an initial speed $v_0$, at a place with latitude $\lambda$ in the northern hemisphere. It reaches a maximum height h, and falls back to the ground. Calculate the magnitude and direction of the Coriolis deflection of the ball when it falls back to the ground. Assume g is unaffected by the centrifugal term.

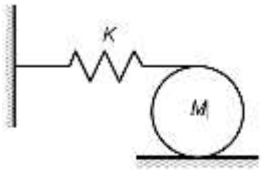
Indian Institute of Science Education and Research Thiruvananthapuram
PHY 312 Classical Mechanics Final Examination (Batch 2009) 18 November 2011, Duration: 3 hours
**Answer any Nine Questions** Marks: 9 x 10 = 90

1. A point mass m is constrained to move on a frictionless plane, fixed to one end of a massless spring of unextended length '$a$' and spring constant $k$ as shown in the figure.



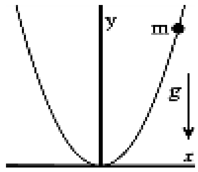
(a) Calculate the total energy of the system and the effective potential.
On a single graph plot the potential and the effective potential under which the particle moves.
(b) What is the angular velocity $\omega_o$ required for a circular orbit of radius $r_o$?
(c) Calculate the frequency of small oscillations $\Omega$ about the circular orbit with radius $r_o$.

2. Consider a homogeneous cylinder of mass M, height h and radius a;
(i) Show that its principal moments of inertia, calculated with the origin at its center of mass and the z axis along its symmetry axis, are $I_{xx} = (1/4)M(a^2+h^2/3) = I_{yy}$ and $I_{zz}=(1/2)Ma^2$.
The cylinder is now constrained to roll on a horizontal surface without slipping and is attached to a spring of stiffness K, as shown in the figure.




(ii) Construct the Lagrangian of the system and determine the equations of motion.
(iii) What happens if the cylinder is disturbed slightly from its equilibrium position? Calculate its natural frequency of small oscillations. Assume that the fixed point of attachment of the spring to the cylinder is at the topmost point at equilibrium.

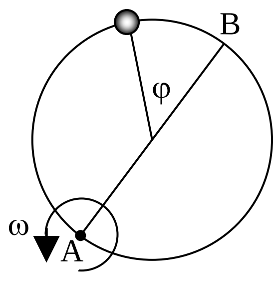
3. A particle of mass m is constrained to move under gravity, along a 'vertical' parabolic trajectory given by $y = x^2/2$. Choose $x$ as the generalised coordinate and construct the Lagrangian of the system. Obtain the Hamiltonian by performing the Legendre transformation of the Lagrangian. Show that the obtained Hamiltonian is the total energy of the system.




4. (i) Two trains travel as follows; one crosses the equator perpendicularly from southern to northern hemisphere and another moves along the equator from east to west. Either train is moving with a speed 200 m/s and each has a plumb line hanging from their ceiling. Calculate the angle made by each plumb line with the local vertical.

(ii) If the experimentally measured value of acceleration due to gravity 'g' at the equator is 10m/s$^2$ what is its value at the north pole? [Earth's radius = 6000 km]

5. A bead of mass m is constrained to move on a massless ring of radius a. The ring rotates with a constant velocity $\omega$ about a vertical axis passing through point A, as shown alongside.
(i) Formulate the Lagrangian of the system with $\varphi$ as the generalised coordinate.
(ii) Show that the bead oscillates about the point B, like a simple pendulum.
(iii) Assuming the amplitude of oscillation is small, calculate the frequency of oscillation.




6. A rigid body rotates freely without any torque. show that in the body fixed axes;
(i) the magnitude of angular momentum **L** is constant.
(ii) the total rotational kinetic energy $T = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ is constant
[$I_i$ : principal moments of inertia, $\omega_i$ : components of angular velocity along the principal axes]
(iii) Show that none of the principal moments of inertia of a rigid body can exceed the sum of other two.

7. (i) In variational calculus if the general integrand $f(y,y',x)$ is independent of $x$ i.e. $f = f(y,y')$ show that;

$$f - y'\frac{\partial f}{\partial y'} = const \text{ where } y' = dy/dx \neq 0.$$

(ii) Say light is travels through an inhomogeneous media, where the refractive index depends only on the height 'y' above the ground and is given by $n(y)$. Show that the path traced by the ray of light always satisfies the condition;

$$\frac{n(y)}{\sqrt{1+y'^2}} = const$$

(iii) If $n(y) = n_o(1+\alpha y)$, ($\alpha$ is constant) solve for the path traced by the ray of light. Simplify the result assuming $\alpha$ is small. [Mention clearly any assumptions made in deriving the expressions].

8. (i) Consider a particle moving in a central force field given by;

$$F(r) = -\frac{k_1}{r^2} - \frac{k_2}{r^4}$$

Show that the condition for a stable circular orbit at $r = r_o$ is, $r_o^2 k_1 > k_2$

(ii) Find the potential for a central force field that allows a particle to move in a spiral orbit given by $r = k\theta^2$.

9. A mass m attached to a massless spring of force constant k, oscillates with a frequency $\omega = \sqrt{(k/m)}$. Assuming the spring also has a mass M distributed uniformly along the length of the spring set up the Hamiltonian of the system. Solve the Hamilton's equation and calculate the new oscillation frequency of the system. [Assume that the spring extends uniformly along its length].

10. A bead of mass m is constrained to slide down a smooth conical spiral, under the action of gravity (see figure). Assume suitable constraint equations in the cylindrical coordinate system and construct the Lagrangian in terms of (r, $\varphi$, z) coordinates and calculate the constraint forces corresponding to each constraint. Using z as the generalised coordinate formulate the Hamiltonian of the system from the Lagrangian.