Assignment 1: Vector Algebra: PHY 121

1. Find the sum or resultant of the following displacements: $\mathbf{A}$, 10 ft northwest; $\mathbf{B}$, 20 ft 30° north of east; $\mathbf{C}$, 35 ft due south.

2. If $x_1\mathbf{a} + y_1\mathbf{b} + z_1\mathbf{c} = x_2\mathbf{a} + y_2\mathbf{b} + z_2\mathbf{c}$, where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-coplanar, then $x_1 = x_2$, $y_1 = y_2$, $z_1 = z_2$.

3. Given $\mathbf{r}_1 = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{r}_2 = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{r}_3 = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, find the magnitudes of (*a*) $\mathbf{r}_3$, (*b*) $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$, (*c*) $2\mathbf{r}_1 - 3\mathbf{r}_2 - 5\mathbf{r}_3$.

4. Find a unit vector parallel to the resultant of vectors $\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, $\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

5. If $\mathbf{A}$ and $\mathbf{B}$ are given vectors show that (*a*) $|\mathbf{A}+\mathbf{B}| \leqq |\mathbf{A}| + |\mathbf{B}|$, (*b*) $|\mathbf{A}-\mathbf{B}| \geqq |\mathbf{A}| - |\mathbf{B}|$.

6. The position vectors of points P and Q are given by $\mathbf{r}_1 = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{r}_2 = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. Determine $\mathbf{PQ}$ in terms of $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and find its magnitude. *Ans.* $2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$, 7

7. If $\mathbf{A} = 3\mathbf{i} - \mathbf{j} - 4\mathbf{k}$, $\mathbf{B} = -2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{C} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, find (*a*) $2\mathbf{A} - \mathbf{B} + 3\mathbf{C}$, (*b*) $|\mathbf{A}+\mathbf{B}+\mathbf{C}|$, (*c*) $|3\mathbf{A} - 2\mathbf{B} + 4\mathbf{C}|$, (*d*) a unit vector parallel to $3\mathbf{A} - 2\mathbf{B} + 4\mathbf{C}$.
Ans. (*a*) $11\mathbf{i} - 8\mathbf{k}$ (*b*) $\sqrt{93}$ (*c*) $\sqrt{398}$ (*d*) $\dfrac{3\mathbf{A} - 2\mathbf{B} + 4\mathbf{C}}{\sqrt{398}}$

8. (*a*) Prove that the vectors $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{B} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{C} = 4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ can form the sides of a triangle.
(*b*) Find the lengths of the medians of the triangle.
Ans. (*b*) $\sqrt{6}$, $\frac{1}{2}\sqrt{114}$, $\frac{1}{2}\sqrt{150}$

9. Graph the vector fields defined by
(*a*) $\mathbf{V}(x,y) = x\mathbf{i} - y\mathbf{j}$, (*b*) $\mathbf{V}(x,y) = y\mathbf{i} - x\mathbf{j}$, (*c*) $\mathbf{V}(x,y,z) = \dfrac{x\mathbf{i} + y\mathbf{i} + z\mathbf{k}}{\sqrt{x^2+y^2+z^2}}$.

10. Find the angle between $\mathbf{A} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

11. If $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, find (*a*) $\mathbf{A}\times\mathbf{B}$, (*b*) $\mathbf{B}\times\mathbf{A}$, (*c*) $(\mathbf{A}+\mathbf{B})\times(\mathbf{A}-\mathbf{B})$.

12. Determine a unit vector perpendicular to the plane of $\mathbf{A} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

13. If $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$, $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$, $\mathbf{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_3\mathbf{k}$ show that

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

14. Find the projection of the vector $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ on the vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. *Ans.* 8/3

15. If $\mathbf{A} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{B} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{C} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, find:
(*a*) $|(\mathbf{A}\times\mathbf{B})\times\mathbf{C}|$, (*c*) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$, (*e*) $(\mathbf{A}\times\mathbf{B})\times(\mathbf{B}\times\mathbf{C})$
(*b*) $|\mathbf{A}\times(\mathbf{B}\times\mathbf{C})|$, (*d*) $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$, (*f*) $(\mathbf{A}\times\mathbf{B})(\mathbf{B}\cdot\mathbf{C})$
Ans. (*a*) $5\sqrt{26}$, (*b*) $3\sqrt{10}$, (*c*) -20, (*d*) -20, (*e*) $-40\mathbf{i} - 20\mathbf{j} + 20\mathbf{k}$, ((*f*) $35\mathbf{i} - 35\mathbf{j} + 35\mathbf{k}$

Assignment 2:

1. Find the Gradients of:
(i) $xz - x^2y + y^2z^2$
(ii) $x^3 + y^3 + z^3$
(iii) $\ln\sqrt{x^2 + y^2 + z^2}$

2. Prove, $\nabla(V^n) = nV^{n-1}\nabla V$.

3. If $\mathbf{A} = xz^3\,\mathbf{i} - 2x^2yz\,\mathbf{j} + 2yz^4\,\mathbf{k}$, find $\nabla\times\mathbf{A}$ (or curl $\mathbf{A}$) at the point $(1,-1,1)$.

4. If $\mathbf{A} = x^2y\,\mathbf{i} - 2xz\,\mathbf{j} + 2yz\,\mathbf{k}$, find curl curl $\mathbf{A}$.

5. Prove that $\nabla\cdot(\nabla\times\mathbf{A}) = \mathbf{0}$.

6. If $\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r}$, prove $\boldsymbol{\omega} = \frac{1}{2}$ curl $\mathbf{v}$ where $\boldsymbol{\omega}$ is a constant vector.

This problem indicates that the curl of a vector field has something to do with rotational properties of the field. This is confirmed in Chapter 6. If the field $\mathbf{F}$ is that due to a moving fluid, for example, then a paddle wheel placed at various points in the field would tend to rotate in regions where curl $\mathbf{F}\neq\mathbf{0}$, while if curl $\mathbf{F}=\mathbf{0}$ in the region there would be no rotation and the field $\mathbf{F}$ is then called *irrotational*. A field which is not irrotational is sometimes called a *vortex field*.

7. (*a*) A vector $\mathbf{V}$ is called irrotational if curl $\mathbf{V}=\mathbf{0}$ (see Problem 30). Find constants a,b,c so that

$$\mathbf{V} = (x + 2y + az)\,\mathbf{i} + (bx - 3y - z)\,\mathbf{j} + (4x + cy + 2z)\,\mathbf{k}$$

is irrotational.

(*b*) Show that $\mathbf{V}$ can be expressed as the gradient of a scalar function.

8. If $\mathbf{A} = 2yz\,\mathbf{i} - x^2y\,\mathbf{j} + xz^2\,\mathbf{k}$, $\mathbf{B} = x^2\mathbf{i} + yz\,\mathbf{j} - xy\,\mathbf{k}$ and $\phi = 2x^2yz^3$, find (*a*) $(\mathbf{A}\cdot\nabla)\phi$, (*b*) $\mathbf{A}\cdot\nabla\phi$, (*c*) $(\mathbf{B}\cdot\nabla)\mathbf{A}$, (*d*) $(\mathbf{A}\times\nabla)\phi$, (*e*) $\mathbf{A}\times\nabla\phi$.

9. If $\nabla U = 2r^4\,\mathbf{r}$, find U.

10. Evaluate $\nabla\cdot(r^3\,\mathbf{r})$. *A*

Evaluate $\nabla\cdot[r\,\nabla(1/r^3)]$

11. If $F = x^2yz$, $G = xy - 3z^2$, find (*a*) $\nabla[(\nabla F)\cdot(\nabla G)]$, (*b*) $\nabla\cdot[(\nabla F)\times(\nabla G)]$, (*c*) $\nabla\times[(\nabla F)\times(\nabla G)]$.

12. For what value of the constant a will the vector $\mathbf{A} = (axy - z^3)\,\mathbf{i} + (a-2)x^2\,\mathbf{j} + (1-a)xz^2\,\mathbf{k}$ have its curl identically equal to zero? *Ans.* $a = 4$

13. If $\mathbf{A} = x^2z\,\mathbf{i} + yz^3\,\mathbf{j} - 3xy\,\mathbf{k}$, $\mathbf{B} = y^2\,\mathbf{i} - yz\,\mathbf{j} + 2x\,\mathbf{k}$ and $\phi = 2x^2 + yz$, find (*a*) $\mathbf{A}\cdot(\nabla\phi)$, (*b*) $(\mathbf{A}\cdot\nabla)\phi$, (*c*) $(\mathbf{A}\cdot\nabla)\mathbf{B}$, (*d*) $\mathbf{B}(\mathbf{A}\cdot\nabla)$, (*e*) $(\nabla\cdot\mathbf{A})\mathbf{B}$.

14. Is there a differentiable vector function $\mathbf{V}$ such that (*a*) curl $\mathbf{V} = \mathbf{r}$, (*b*) curl $\mathbf{V} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$? If so, find $\mathbf{V}$.

Assignment 3: Vector Integrals, Divergence and Stokes' Theorem, Curvilinear Coordinates

1. (*a*) If $\mathbf{F} = \nabla\phi$, where ϕ is single-valued and has continuous partial derivatives, show that the work done in moving a particle from one point $P_1 \equiv (x_1, y_1, z_1)$ in this field to another point $P_2 \equiv (x_2, y_2, z_2)$ is independent of the path joining the two points.

   (*b*) Conversely, if $\int_C \mathbf{F}\cdot d\mathbf{r}$ is independent of the path C joining any two points, show that there exists a function ϕ such that $\mathbf{F} = \nabla\phi$.

2. (*a*) If $\mathbf{F}$ is a conservative field, prove that $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \mathbf{0}$ (i.e. $\mathbf{F}$ is irrotational).
   (*b*) Conversely, if $\nabla\times\mathbf{F} = \mathbf{0}$ (i.e. $\mathbf{F}$ is irrotational), prove that $\mathbf{F}$ is conservative.

3. (*a*) Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. (*b*) Find the scalar potential. (*c*) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

4. Evaluate $\iint_S \mathbf{A}\cdot\mathbf{n}\, dS$, where $\mathbf{A} = z\,\mathbf{i} + x\,\mathbf{j} - 3y^2 z\,\mathbf{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

5. If $\mathbf{F} = y\,\mathbf{i} + (x - 2xz)\mathbf{j} - xy\,\mathbf{k}$, evaluate $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane.

6. Let $\mathbf{F} = 2xz\,\mathbf{i} - x\,\mathbf{j} + y^2\mathbf{k}$. Evaluate $\iiint_V \mathbf{F}\, dV$ where V is the region bounded by the surfaces $x = 0$, $y = 0$, $y = 6$, $z = x^2$, $z = 4$.

7. (*a*) Prove that $\mathbf{F} = (y^2\cos x + z^3)\mathbf{i} + (2y\sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$ is a conservative force field.
   (*b*) Find the scalar potential for $\mathbf{F}$.
   (*c*) Find the work done in moving an object in this field from $(0,1,-1)$ to $(\pi/2,-1,2)$.

8. Evaluate $\iint_S \mathbf{A}\cdot\mathbf{n}\, dS$ over the entire surface of the region above the xy plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, if $\mathbf{A} = 4xz\,\mathbf{i} + xyz^2\,\mathbf{j} + 3z\,\mathbf{k}$. *Ans.* 320π

9. Evaluate $\iint \sqrt{x^2+y^2}\, dx\, dy$ over the region R in the xy plane bounded by $x^2 + y^2 = 36$. *Ans.* 144π

10. Evaluate $\iiint_V (2x + y)\, dV$, where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 2$ and $z = 0$. *Ans.* $80/3$

11. Verify Stokes' theorem for $\mathbf{A} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2 z\,\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

12. Show that in polar coordinates (ρ, ϕ) the expression $x\,dy - y\,dx = \rho^2\, d\phi$. Interpret $\frac{1}{2}\int x\,dy - y\,dx$.

13. In the Spherical coordinate system plot the curve given by $\theta = \pi/3$.

14. Represent the vector $\mathbf{A} = z\mathbf{i} - 2x\mathbf{j} + y\mathbf{k}$ in cylindrical coordinates. Thus determine A_ρ, A_ϕ and A_z.

15. (*a*) Find the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ of a spherical coordinate system in terms of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$.
    (*b*) Solve for $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ in terms of $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.

16. For the function $z(x,y) = (x^2 - y^2)e^{-x2-y2}$ find the location(s) at which the steepest gradient occurs. What are the magnitude and direction of that gradient? The algebra involved is easier if plane polar coordinates are used.

17. In the following exercises **a** is a vector field.
(a) Simplify $\nabla\times \mathbf{a}(\nabla\cdot \mathbf{a}) + \mathbf{a} \times [\nabla\times (\nabla\times \mathbf{a})] + \mathbf{a} \times \nabla^2\mathbf{a}$
(b) By explicitly writing out the terms in Cartesian coordinates prove that $[\mathbf{c} \cdot (\mathbf{b} \cdot \nabla) - \mathbf{b} \cdot (\mathbf{c} \cdot \nabla)]\, \mathbf{a} = (\nabla\times \mathbf{a}) \cdot (\mathbf{b} \times \mathbf{c})$
(c) Prove that $\mathbf{a} \times (\nabla\times \mathbf{a}) = \nabla(1/2\, a^2) - (\mathbf{a} \cdot \nabla)\mathbf{a}$

18. The vector field **Q** is defined as $\mathbf{Q} = (3x^2(y + z) + y^3 + z^3)\mathbf{i} + (3y^2(z + x) + z^3 + x^3)\mathbf{j} + (3z^2(x + y) + x^3 + y^3)\mathbf{k}$.
Show that **Q** is a conservative field and construct its scalar function and hence evaluate the integral $J = \mathbf{Q}\cdot d\mathbf{r}$ along any line connecting the point A at (1,−1, 1)to B at (2, 1, 2). Hint: Show $\nabla\times \mathbf{Q} = 0$ to illustrate a conservative force field

Assignment 4: PHY 121: Electrostatics: Discrete Charges

3.1. Calculate the ratio of the electrostatic to gravitational interaction forces between two electrons, between two protons. At what value of the specific charge q/m of a particle would these forces become equal (in their absolute values) in the case of interaction of identical particles?
What can you conclude about the relative strengths of the two forces?

3.4. Two positive charges q_1 and q_2 are located at the points with radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. Find a negative charge q_3 and a radius vector $\mathbf{r}_3$ of the point at which it has to be placed for the force acting on each of the three charges to be equal to zero.

3.6. A positive point charge 50 μC is located in the plane xy at the point with radius vector $\mathbf{r}_0 = 2\mathbf{i} + 3\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find the vector of the electric field strength $\mathbf{E}$ and its magnitude at the point with radius vector $\mathbf{r} = 8\mathbf{i} - 5\mathbf{j}$. Here r_0 and r are expressed in metres.

3.7. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in Fig. 3.1. Find the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from its centre.

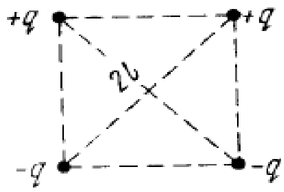


Fig. 3.1.

1. *Electric field along the line passing through two point charges.*
A point charge $Q_1 = +3\ \mu$C is placed at the origin, and a point charge $Q_2 = -7\ \mu$C is placed at $x = 0.4$ m on the x-axis of a cartesian coordinate system.

   (a) Determine the electric field, $\vec{E}(x) = E(x)\hat{x}$, at all points along the x-axis.

   (b) Plot $E(x)$ vs. x for $x < 0$.

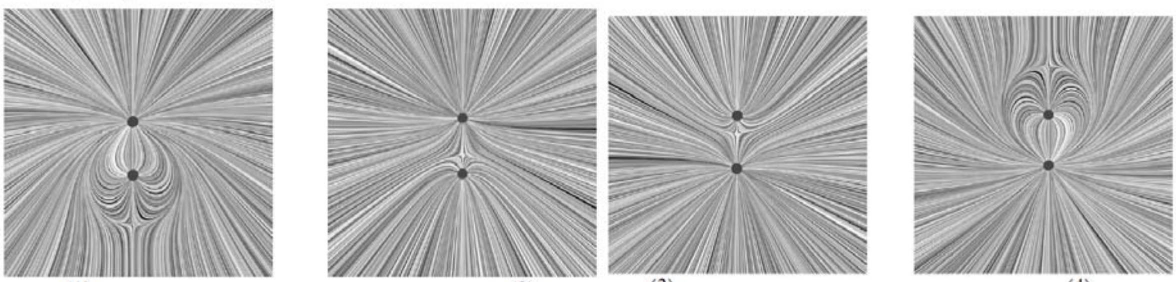
   (c) At what points, if any, (apart from $|x| = \infty$), is $E(x) = 0$?

2. Two objects with charges $-q$ and $+3q$ are placed on a line as shown in the figure below.

Besides an infinite distance away from the charges, where else can the electric field possibly be zero? **Explain your reasoning.**

1. Between the two charges.
2. To the right of the charge on the right.
3. To the left of the charge on the left.
4. The electric field is only zero an infinite distance away
5. from the charges.

3. **(b) (5 points).** Two objects with charges $-4Q$ and $-Q$ lie on the y-axis. The object with the charge $-4Q$ is ***above*** the object with charge $-Q$. Below are four possible "grass seed" representations of the electric field of the two charges. Which of these representations is most nearly right for the two charges in this problem? **Explain your reasoning.**

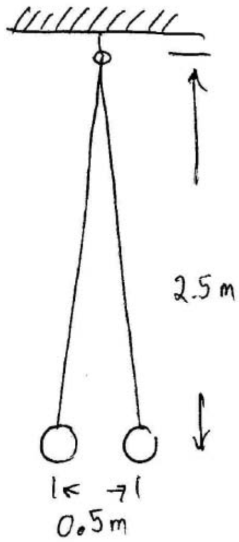


(1) (2) (3) (4)

4. Calculate the ratio of the electric force and gravitational force between a single proton and a single electron (a hydrogen atom); assuming a classical model in which the electron revolves around the proton in a circular orbit.

Problem 5: Coulomb's Law (10 points)

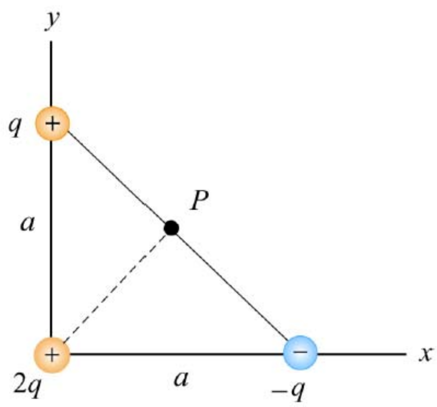
Two volley balls, each of mass $m = 0.2\text{ kg}$, tethered by nylon strings and equally charged with an electrostatic generator, hang as shown in the figure such that the centers of the balls are a distance $r = 0.5\text{ m}$ apart. The point equidistance between the two centers of the balls is a distance $d = 2.5\text{ m}$ below the suspension point. What is the charge on each ball? Include your free-body force diagram in your solution.



Problem 6 Electric field for a Distribution of Point Charges (10 points)

A right isosceles triangle of side a has objects with charges q, $+2q$ and $-q$ arranged on its vertices.

What is the magnitude and direction of the electric field at point P due to the charges in the figure, midway between the line connecting the $+q$ and $-q$ charges?



Assignment 5:

3.11. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

3.17. Suppose the surface charge density over a sphere of radius R depends on a polar angle θ as $\sigma = \sigma_0 \cos\theta$, where σ_0 is a positive constant. Show that such a charge distribution can be represented as a result of a small relative shift of two uniformly charged *balls* of radius R whose charges are equal in magnitude and opposite in sign. Resorting to this representation, find the electric field strength vector inside the given sphere.

3.20. Two point charges q and $-q$ are separated by the distance $2l$ (Fig. 3.3). Find the flux of the electric field strength vector across a circle of radius R.

3.27. A space is filled up with a charge with volume density $\rho = \rho_0 e^{-\alpha r^3}$, where ρ_0 and α are positive constants, r is the distance from the centre of this system. Find the magnitude of the electric field strength vector as a function of r. Investigate the obtained expression for the small and large values of r, i.e. at $\alpha r^3 \ll 1$ and $\alpha r^3 \gg 1$.

3.53. The field potential inside a charged ball depends only on the distance from its centre as $\varphi = ar^2 + b$, where a and b are constants. Find the space charge distribution $\rho(r)$ inside the ball.

3.52. A uniformly distributed space charge fills up the space between two large parallel plates separated by a distance d. The potential difference between the plates is equal to $\Delta\varphi$. At what value of charge density ρ is the field strength in the vicinity of one of the plates equal to zero? What will then be the field strength near the other plate?

3.61. A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance l. The thread carries a uniform charge of linear density λ. Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane
(a) at the point O;
(b) as a function of a distance r from the point O.

3.137. A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2. Find the work performed by the electric forces in this process.

The following problems from Griffiths:
2.40, 2.44, 2.47, 3.6, 3.10, 3.11.

Assignment 6:

3.40. A point dipole with an electric moment p oriented in the positive direction of the z axis is located at the origin of coordinates. Find the projections E_z and $E_\perp$ of the electric field strength vector (on the plane perpendicular to the z axis at the point S (see Fig. 3.4)). At which points is $\mathbf{E}$ perpendicular to $\mathbf{p}$?

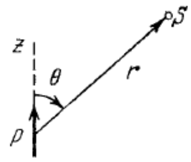


Fig. 3.4.

3.42. Two thin parallel threads carry a uniform charge with linear densities λ and $-\lambda$. The distance between the threads is equal to l. Find the potential of the electric field and the magnitude of its strength vector at the distance $r \gg l$ at the angle θ to the vector $\mathbf{l}$ (Fig. 3.5).

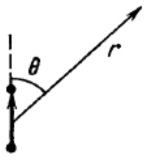


Fig. 3.5.

3.54. A small ball is suspended over an infinite horizontal conducting plane by means of an insulating elastic thread of stiffness k. As soon as the ball was charged, it descended by x cm and its separation from the plane became equal to l. Find the charge of the ball.

3.59. A point charge q is located at a distance l from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation r from the base of the perpendicular drawn to the plane from the charge.

3.66. Four large metal plates are located at a small distance d from one another as shown in Fig. 3.8. The extreme plates are inter-

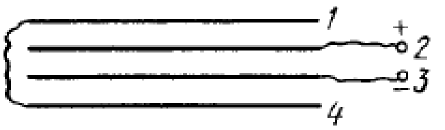


Fig. 3.8.

connected by means of a conductor while a potential difference $\Delta\varphi$ is applied to internal plates. Find:

(a) the values of the electric field strength between neighbouring plates;

(b) the total charge per unit area of each plate.

3.70. When an uncharged conducting ball of radius R is placed in an external uniform electric field, a surface charge density $\sigma = \sigma_0 \cos\theta$ is induced on the ball's surface (here σ_0 is a constant, θ is a polar angle). Find the magnitude of the resultant electric force acting on an induced charge of the same sign.

3.72. A non-polar molecule with polarizability β is located at a great distance l from a polar molecule with electric moment $\mathbf{p}$. Find the magnitude of the interaction force between the molecules if the vector $\mathbf{p}$ is oriented along a straight line passing through both molecules.

3.75. Demonstrate that at a dielectric-conductor interface the surface density of the dielectric's bound charge $\sigma' = -\sigma(\varepsilon - 1)/\varepsilon$, where ε is the permittivity, σ is the surface density of the charge on the conductor.

3.78. Near the point A (Fig. 3.10) lying on the boundary between glass and vacuum the electric field strength in vacuum is equal to $E_0 = 10.0$ V/m, the angle between the vector $\mathbf{E}_0$ and the normal $\mathbf{n}$ of the boundary line being equal to $\alpha_0 = 30°$. Find the field strength E in glass near the point A, the angle α between the vector $\mathbf{E}$ and $\mathbf{n}$, as well as the surface density of the bound charges at the point A.

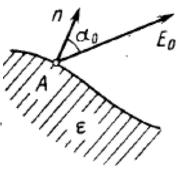


Fig. 3.10.

3.83. Under certain conditions the polarization of an infinite uncharged dielectric plate takes the form $\mathbf{P} = \mathbf{P}_0(1 - x^2/d^2)$, where $\mathbf{P}_0$ is a vector perpendicular to the plate, x is the distance from the middle of the plate, d is its half-thickness. Find the strength $\mathbf{E}$ of the electric field inside the plate and the potential difference between its surfaces.

3.87. Two small identical balls carrying the charges of the same sign are suspended from the same point by insulating threads of equal length. When the surrounding space was filled with kerosene the divergence angle between the threads remained constant. What is the density of the material of which the balls are made?
Given that the density of kerosene and the material of the balls are ρ' and ρ respectively and permittivity of kerosene is ε'.

3.99. An infinitely long round dielectric cylinder is polarized uniformly and statically, the polarization $\mathbf{P}$ being perpendicular to the axis of the cylinder. Find the electric field strength $\mathbf{E}$ inside the dielectric.

3.101. Find the capacitance of an isolated ball-shaped conductor of radius R_1 surrounded by an adjacent concentric layer of dielectric with permittivity ε and outside radius R_2.

3.104. The gap between the plates of a parallel-plate capacitor is filled with isotropic dielectric whose permittivity ε varies linearly from ε_1 to ε_2 ($\varepsilon_2 > \varepsilon_1$) in the direction perpendicular to the plates. The area of each plate equals S, the separation between the plates is equal to d. Find:

(a) the capacitance of the capacitor;

(b) the space density of the bound charges as a function of ε if the charge of the capacitor is q and the field $\mathbf{E}$ in it is directed toward the growing ε values.

3.109. A long straight wire is located parallel to an infinite conducting plate. The wire cross-sectional radius is equal to a, the distance between the axis of the wire and the plane equals b. Find the mutual capacitance of this system per unit length of the wire under the condition $a \ll b$.

Fig. 3.16.

3.140. A point charge q is located at the centre O of a spherical uncharged conducting layer provided with a small orifice (Fig. 3.32). The inside and outside radii of the layer are equal to a and b respectively. What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and into infinity?

Fig. 3.32.

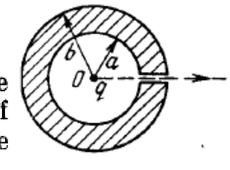


Fig. 3.32.

3.144. A parallel-plate capacitor is located horizontally so that one of its plates is submerged into liquid while the other is over its surface (Fig. 3.33). The permittivity of the liquid is equal to ε, its density is equal to ρ. To what height will the level of the liquid in the capacitor rise after its plates get a charge of surface density σ?

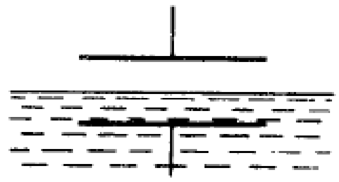


Fig. 3.33

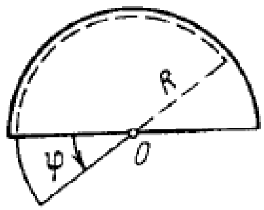


Fig. 3.34.

3.145. A cylindrical layer of dielectric with permittivity ε is inserted into a cylindrical capacitor to fill up all the space between the electrodes. The mean radius of the electrodes equals R, the gap between them is equal to d, with $d \ll R$. The constant voltage V is applied across the electrodes of the capacitor. Find the magnitude of the electric force pulling the dielectric into the capacitor.

3.146. A capacitor consists of two stationary plates shaped as a semi-circle of radius R and a movable plate made of dielectric with permittivity ε and capable of rotating about an axis O between the stationary plates (Fig. 3.34). The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in the figure.

3.141. Each plate of a parallel-plate air capacitor has an area S. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if

(a) the capacitance of the capacitor, which is equal to q, or (b) the voltage across the capacitor, which is equal to V, is kept constant in the process?

3.220. A current I flows along a thin wire shaped as a regular polygon with n sides which can be inscribed into a circle of radius R. Find the magnetic induction at the centre of the polygon. Analyse the obtained expression at $n \to \infty$.

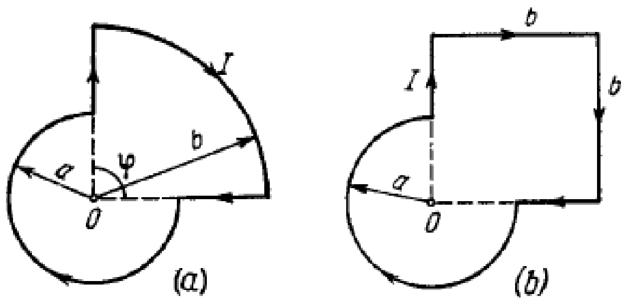


Fig. 3.60.

3.223. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated

(a) in Fig. 3.60a, the radii a and b, as well as the angle φ are known;

(b) in Fig. 3.60b, the radius a and the side b are known.

3.224. A current I flows along a lengthy thin-walled tube of radius R with longitudinal slit of width h. Find the induction of the magnetic field inside the tube under the condition $h \ll R$.

Note: we can use the principle of superposition to solve this problem, can you figure out how?

3.232. A current I flows along a round loop. Find the integral $\int \mathbf{B}\, d\mathbf{r}$ along the axis of the loop within the range from $-\infty$ to $+\infty$. Explain the result obtained.

3.236. A single-layer coil (solenoid) has length l and cross-section radius R. A number of turns per unit length is equal to n. Find the magnetic induction at the centre of the coil when a current I flows through it.

5. A proton moves perpendicular to a uniform magnetic field $\mathbf{B}$ at 1.00×10^7 m/s and experiences an acceleration of 2.00×10^{13} m/s^2 in the $+x$ direction when its velocity is in the $+z$ direction. Determine the magnitude and direction of the field.

7. A proton moving at 4.00×10^6 m/s through a magnetic field of 1.70 T experiences a magnetic force of magnitude 8.20×10^{-13} N. What is the angle between the proton's velocity and the field?

40. A velocity selector consists of electric and magnetic fields described by the expressions $\mathbf{E} = E\hat{\mathbf{k}}$ and $\mathbf{B} = B\hat{\mathbf{j}}$, with $B = 15.0$ mT. Find the value of E such that a 750-eV electron moving along the positive x axis is undeflected.

34. **Review Problem.** An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1.00 mT. The angular momentum of the electron about the center of the circle is 4.00×10^{-25} J·s. Determine (a) the radius of the circular path and (b) the speed of the electron.

35. Calculate the cyclotron frequency of a proton in a magnetic field of magnitude 5.20 T.

$\boxed{1.}$ Determine the initial direction of the deflection of charged particles as they enter the magnetic fields as shown in Figure P29.1.

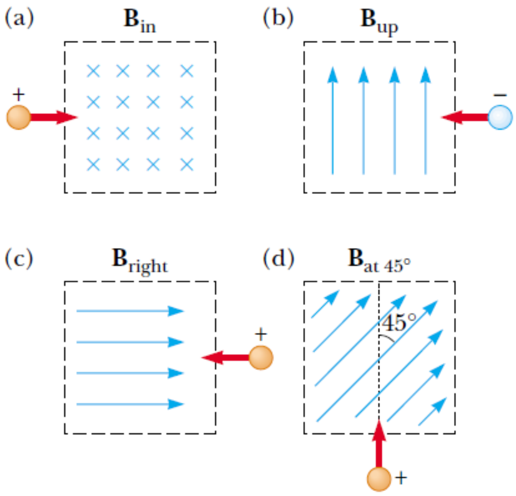


Figure P29.1

66. A uniform magnetic field of magnitude 0.150 T is directed along the positive x axis. A positron moving at 5.00×10^6 m/s enters the field along a direction that makes an angle of 85.0° with the x axis (Fig. P29.66). The motion of the particle is expected to be a helix, as described in Section 29.4. Calculate (a) the pitch p and (b) the radius r of the trajectory.

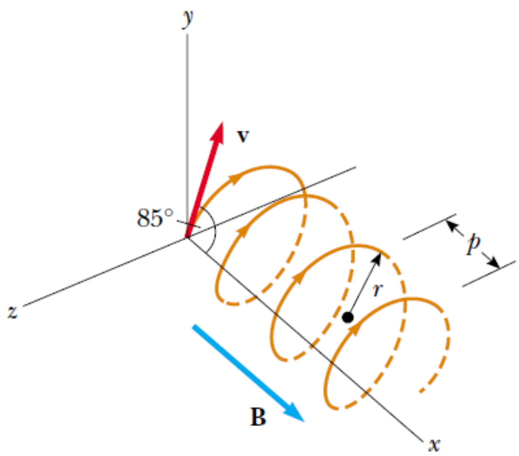


Figure P29.66

Assignment 8: Image Charges

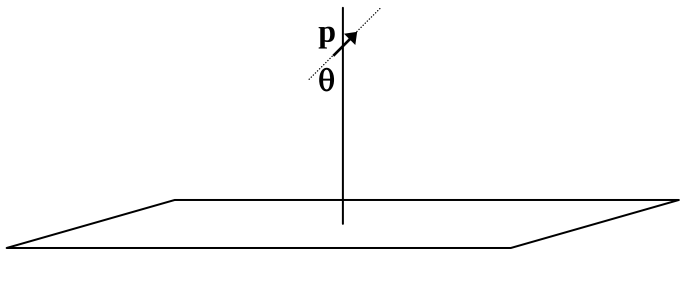
1. A total charge Q is uniformly distributed along a circular ring of radius b. The centre of the ring is located at (0,0,d) in cylindrical coordinate system. Assuming that the plane of the ring is parallel to a perfectly conducting horizontal plane located at z = 0, calculate the electric field and potential at any point along the axis of the ring. What is the work done in bringing another point charge q and placed at the point $(0,0,h_1)$?

2. 4 point charges q, -q, q, -q are located at the corners of a square of side a, with like charges diagonally opposite each other. Show that there are 2 equipotential surfaces that are planes; each plane contains one diagonal and are perpendicular to each other. How will the orientation of the equipotentials change if one side of the square is stretched to length b, making the charge distribution rectangular?

3. With reference to the above problem, find the position(s) of the image charges for a single point charge placed somewhere in the inside corner formed by bending, an infinite conducting sheet, at an angle $\theta = 90$ deg. Specifically reason that the method of images will not work for any angles between $180 > \theta > 90$. Also reason that the method of images only works for angles $\theta = 180/N$, where N is an integer. Thus $\theta =$ 180, 90, 45, 22.5 ..... . Why does this technique not work for other/arbitrary angles?

4. 2 infinitely long wires running parallel to the x axis carry uniform charge densities $+\lambda$ and $-\lambda$. they are located at $y = \pm a$, on the xy plane. Write an expression for the potential at an arbitrary point (x,y,z). Show that the equipotentials are surfaces of circular cylinders. Can you construct an equivalent image problem that that might be solved, keeping in mind the shape of the equipotential surfaces.

5. An electric dipole of dipole moment **p** is kept at a distance h above an infinitely large conducting plane (say the xy plane), as shown below. Calculate the potential and electric field over all space.



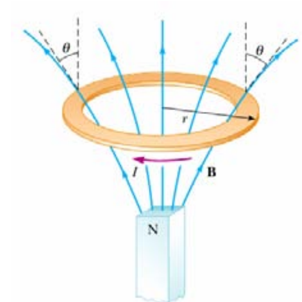
PHY 121 Assignment 9:

1. In an experiment that is designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east–west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10×10^{-12} V, what is the magnitude of the Earth's magnetic field? (Assume that n = 8.49×10^{28} electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

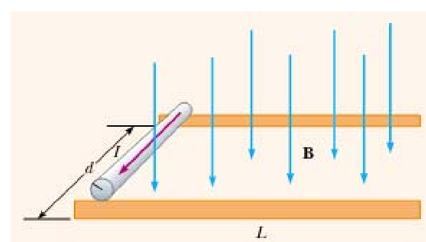
2. A Hall-effect probe operates with a 120-mA current. When the probe is placed in a uniform magnetic field of magnitude 0.0800 T, it produces a Hall voltage of 0.700 ìV.
(a) When it is measuring an unknown magnetic field, the Hall voltage is 0.330 ìV. What is the magnitude of the unknown field?
(b) The thickness of the probe in the direction of B is 2.00 mm. Find the density of the charge carriers, each of which has charge of magnitude e.

3. (a) A proton moving in the +x direction with velocity **v** = v_i î experiences a magnetic force **F**=F_i j in the +y direction. Explain what you can and cannot infer about **B** from this information. (b) What If? In terms of F_i, what would be the force on a proton in the same field moving with velocity **v** = ? v_i î? (c) What would be the force on an electron in the same field moving with velocity **v** = ? v_i î?

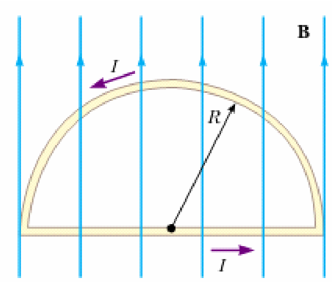
4. A nonuniform magnetic field exerts a net force on a magnetic dipole. A strong magnet is placed under a horizontal conducting ring of radius r that carries current I as shown in Figure. If the magnetic field **B** makes an angle è with the vertical at the ring's location, what are the magnitude and direction of the resultant force on the ring?



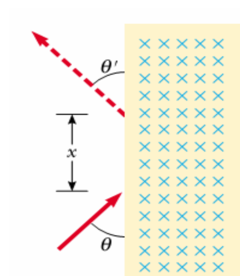
5. A rod of mass 0.720 kg and radius 6.00 cm rests on two parallel rails that are d = 12.0 cm apart and L = 45.0 cm long. The rod carries a current of I = 48.0 A (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field of magnitude 0.240 T is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails?



6. A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis, as shown in Figure. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.

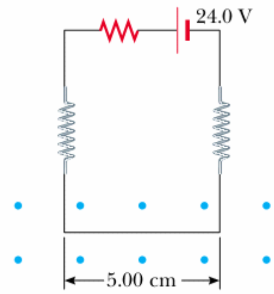


7. A proton moving in the plane of the page has a kinetic energy of 6.00 MeV. A magnetic field of magnitude B = 1.00 T is directed into the page. The proton enters the magnetic field with its velocity vector at an angle è = 45.0 ° to the linear boundary of the field as shown in Figure. (a) Find x, the distance from the point of entry to where the proton will leave the field. (b) Determine è', the angle between the boundary and the proton's velocity vector as it leaves the field.

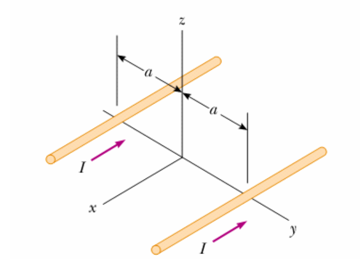


8. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^{6} m/s. (a) What is the magnitude of the magnetic moment due to the electron's motion? (b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of this magnetic moment vector?

9. The circuit in Figure consists of wires at the top and bottom and identical metal springs in the left and right sides. The upper portion of the circuit is fixed. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The springs stretch 0.500 cm under the weight of the wire and the circuit has a total resistance of 12.0 ?. When a magnetic field is turned on, directed out of the page, the springs stretch an additional 0.300 cm. What is the magnitude of the magnetic field?



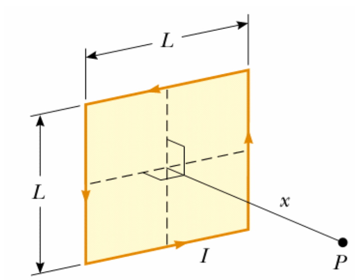
10. In Figure, both currents in the in finitely long wires are in the negative x direction. (a) Sketch the magnetic field pattern in the yz plane. (b) At what distance d along the z axis is the magnetic field a maximum?



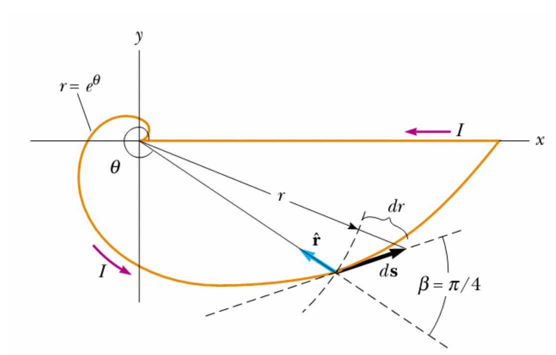
11. What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

12. A wire is formed into the shape of a square of edge length L. Show that when the current in the loop is I, the magnetic field at point P, a distance x from the center of the square along its axis is

$$B = \frac{\mu_0 I L^2}{2\pi(x^2 + L^2/4)\sqrt{x^2 + L^2/2}}$$



13. A wire carrying a current I is bent into the shape of an exponential spiral, r = e^{è} , from è = 0 to è = 2ð as suggested in Figure. To complete a loop, the ends of the spiral are connected by a straight wire along the x axis. Find the magnitude and direction of **B** at the origin.

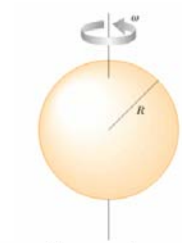


Suggestions: Use the Biot–Savart law. The angle â between a radial line and its tangent line at any point on the curve r = f (è) is related to the function in the following way: $\tan\beta = \dfrac{r}{dr/d\theta}$

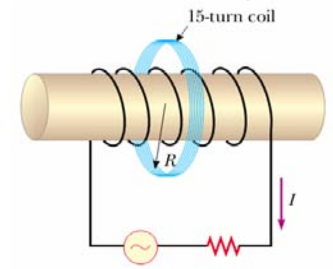
Thus in this case r = e^{è} , tanâ = 1 and â = ð/4. Therefore, the angle between **ds** and **r** is ð ? â= 3ð/4. Also

$$ds = \frac{dr}{\sin(\pi/4)} = \sqrt{2}\ dr$$

14. A sphere of radius R has a uniform volume charge density ñ. Determine the magnetic field at the center of the sphere when it rotates as a rigid object with angular speed ù about an axis through its center.



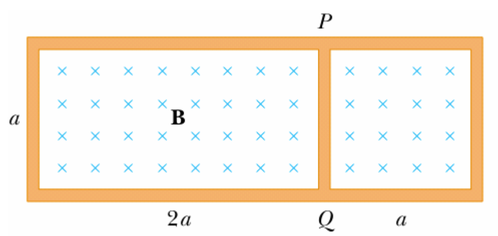
15. A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.00 cm and 1.00×10^{3} turns/meter. The current in the solenoid changes as I = (5.00 A) sin (120t). Find the induced emf in the 15-turn coil as a function of time.



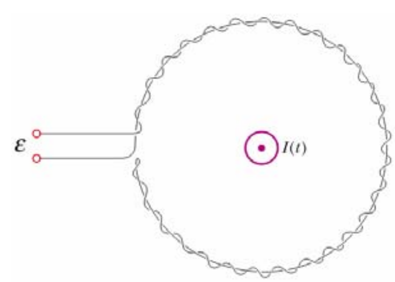
16. A rectangular loop of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to B = B_{max} $e^{-t/ô}$, where B_{max} and ô are constants. The field has the constant value B_{max} for t < 0.
(a) Use Faraday's law to show that the emf induced in the loop is given by $$\varepsilon = \frac{AB_{max}}{\tau} e^{-t/\tau}$$

(b) Obtain a numerical value for **å** at t = 4.00 s when A = 0.160 m^2, B_{max} = 0.350 T, and ô = 2.00 s. (c) For the values of A, B_{max}, and ô given in (b), what is the maximum value of **å**?

17. Find the current through section PQ of length a = 65.0 cm in Figure. The circuit is located in a magnetic field whose magnitude varies with time according to the expression B = $(1.00 \times 10^{-3}$ T/s)t. Assume the resistance per length of the wire is 0.100 ?/m.

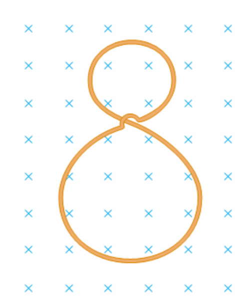


18. When a wire carries an AC current with a known frequency, you can use a Rogowski coil to determine the amplitude I_{max} of the current without disconnecting the wire to shunt the current in a meter. The Rogowski coil, shown in Figure, simply clips around the wire. It consists of a toroidal conductor wrapped around a circular return cord. The toroid has n turns per unit length and a cross-sectional area A. The current to be measured is given by I (t) =I_{max} sin ùt. (a) Show that the amplitude of the emf induced in the Rogowski coil is $å_{max}$ = $ì_0$nAù I_{max}. (b) Explain why the wire carrying the unknown current need not be at the center of the Rogowski coil, and why the coil will not respond to nearby currents that it does not enclose.

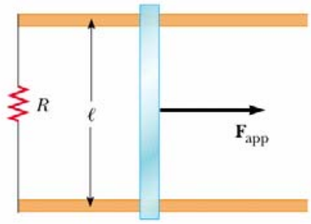


19. A long solenoid has n = 400 turns per meter and carries a current given by I = (30.0 A)(1 ? $e^{-1.60\,t}$). Inside the solenoid and coaxial with it is a coil that has a radius of 6.00 cm and consists of a total of N = 250 turns of fine wire. What emf is induced in the coil by the changing current?

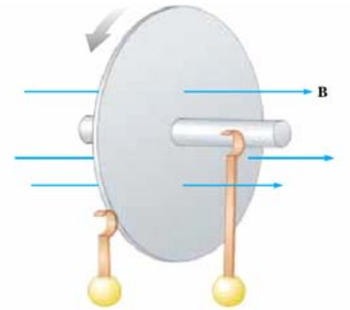
20. A piece of insulated wire is shaped into a figure 8, as in Figure. The radius of the upper circle is 5.00 cm and that of the lower circle is 9.00 cm. The wire has a uniform resistance per unit length of 3.00 ?/m. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown. The magnetic field is increasing at a constant rate of 2.00 T/s. Find the magnitude and direction of the induced current in the wire.

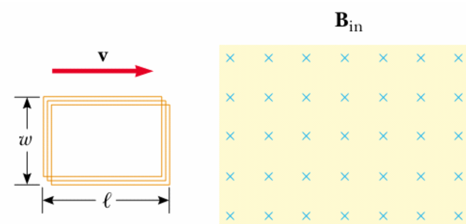


21. Figure shows a top view of a bar that can slide without friction. The resistor is 6.00 ? and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let ? = 1.20 m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is energy delivered to the resistor?



22. The homopolar generator, also called the Faraday disk, is a low voltage, high-current electric generator. It consists of a rotating conducting disk with one stationar y brush (a sliding electrical contact) at its axle and another at a point on its circumference, as shown in Figure. A magnetic field is applied perpendicular to the plane of the disk. Assume the field is 0.900 T, the angular speed is 3200 rev/min, and the radius of the disk is 0.400 m. Find the emf generated between the brushes. When superconducting coils are used to produce a large magnetic field, a homopolar generator can have a power output of several megawatts. Such a generator is useful, for example, in purifying metals by electrolysis. If a voltage is applied to the output terminals of the generator, it runs in reverse as a homopolar motorcapable of providing great torque, useful in ship propulsion.





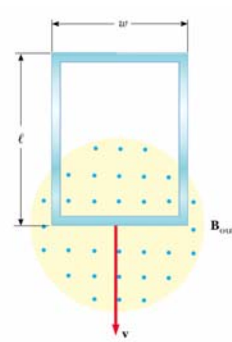
23. An automobile has a vertical radio antenna 1.20 m long. The automobile travels at 65.0 km/h on a horizontal road where the Earth's magnetic field is 50.0 /T directed toward the north and downward at an angle of 65.0° below the horizontal. (a) Specify the direction that the automobile should move in order to generate the maximum motional emf in the antenna, with the top of the antenna positive relative to the bottom. (b) Calculate the magnitude of this induced emf.

24. A rectangular coil with resistance R has N turns, each of length l and width w as shown in Figure. The coil moves into a uniform magnetic field B with constant velocity v. What are the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field, (b) as it moves within the field, and (c) as it leaves the field?

25. A long solenoid with 1000 turns per meter and radius 2.00 cm carries an oscillating current given by I = (5.00 A) sin(100ðt). What is the electric field induced at a radius r = 1.00 cm from the axis of the solenoid? What is the direction of this electric field when the current is increasing counterclockwise in the coil?

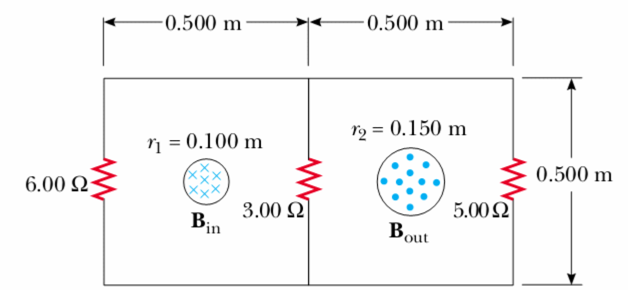
26. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field **B** as shown in Figure. During the time interval before the top edge of the loop reaches the field, the loop approaches a terminal speed v_T. (a) Show that

$$v_T = \frac{MgR}{B^2 w^2}$$



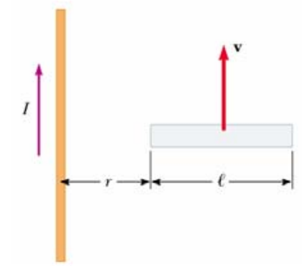
(b) Why is v_T proportional to R? (c) Why is it inversely proportional to B^2?

7. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Figure. The magnitude of **B** inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?



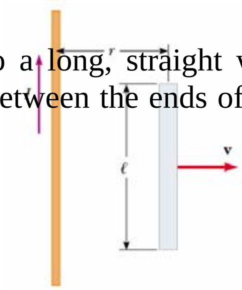
28. A conducting rod of length *l* moves with velocity v parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away, as shown in Figure. Show that the magnitude of the emf induced in the rod is

$$|\mathcal{E}| = \frac{\mu_0 I v}{2\pi} \ln\left(1 + \frac{\ell}{r}\right)$$



29. A conducting rod moves with a constant velocity **v** in a direction perpendicular to a long, straight wire carrying a current I as shown in Figure. Show that the magnitude of the emf generated between the ends of the rod is

$$|\mathcal{E}| = \frac{\mu_0 v I \ell}{2\pi r}$$

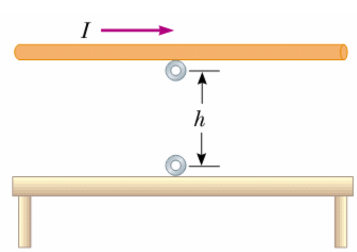


In this case, note that the emf decreases with increasing r, as you might expect.

30. A particle with a mass of 2.00×10^{-16} kg and a charge of 30.0 nC starts from rest, is accelerated by a strong electric field, and is fired from a small source inside a region of uniform constant magnetic field 0.600 T. The velocity of the particle is perpendicular to the field. The circular orbit of the particle encloses a magnetic flux of 15.0 ìWb. (a) Calculate the speed of the particle. (b) Calculate the potential difference through which the particle accelerated inside the source.

31. A betatron accelerates electrons to energies in the MeV range by means of electromagnetic induction. Electrons in a vacuum chamber are held in a circular orbit by a magnetic field perpendicular to the orbital plane. The magnetic field is gradually increased to induce an electric field around the orbit. (a) Show that the electric field is in the correct direction to make the electrons speed up. (b)Assume that the radius of the orbit remains constant. Show that the average magnetic field over the area enclosed by the orbit must be twice as large as the magnetic field at the circumference of the circle.

32. A small circular washer of radius 0.500 cm is held directly below a long, straight wire carrying a current of 10.0 A. The washer is located 0.500 m above the top of a table. (a) If the washer is dropped from rest, what is the magnitude of the average induced emf in the washer from the time it is released to the moment it hits the tabletop? Assume that the magnetic field is nearly constant over the area of the washer, and equal to the magnetic field at the center of the washer. (b) What is the direction of the induced current in the washer?



Assignment 9:

1. In an experiment that is designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east–west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10×10^{-12} V, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.49 \times 10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

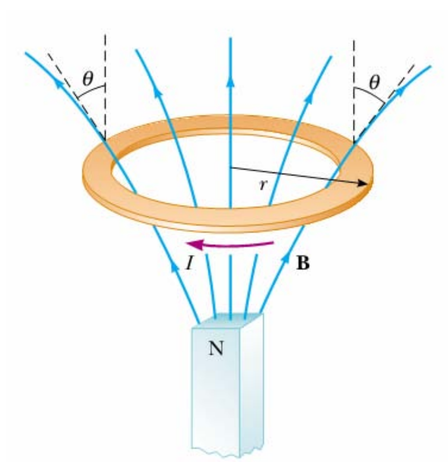
2. A Hall-effect probe operates with a 120-mA current. When the probe is placed in a uniform magnetic field of magnitude 0.0800 T, it produces a Hall voltage of 0.700 μV.

(a) When it is measuring an unknown magnetic field, the Hall voltage is 0.330 μV. What is the magnitude of the unknown field?

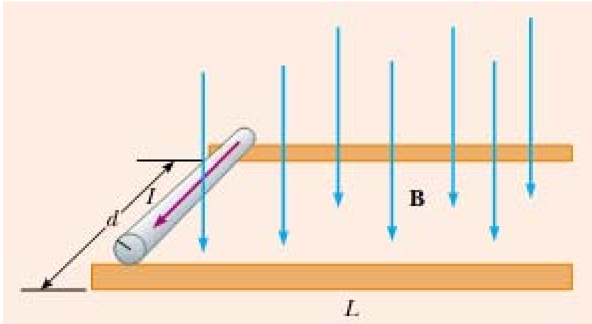
(b) The thickness of the probe in the direction of B is 2.00 mm. Find the density of the charge carriers, each of which has charge of magnitude e.

3. (a) A proton moving in the +x direction with velocity $\mathbf{v} = v_i\,\hat{i}$ experiences a magnetic force $\mathbf{F}=F_i\,\hat{j}$ in the +y direction. Explain what you can and cannot infer about **B** from this information. (b) What If? In terms of F_i, what would be the force on a proton in the same field moving with velocity $\mathbf{v} = -v_i\,\hat{i}$? (c) What would be the force on an electron in the same field moving with velocity $\mathbf{v} = -v_i\,\hat{i}$?

4. A nonuniform magnetic field exerts a net force on a magnetic dipole. A strong magnet is placed under a horizontal conducting ring of radius r that carries current I as shown in Figure. If the magnetic field **B** makes an angle θ with the vertical at the ring's location, what are the magnitude and direction of the resultant force on the ring?

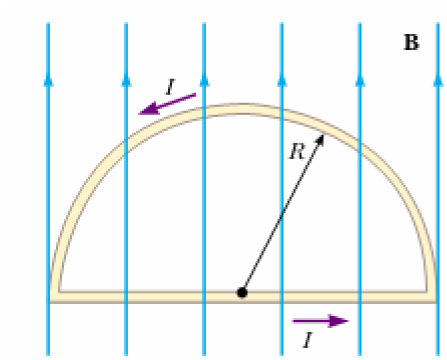


5. A rod of mass 0.720 kg and radius 6.00 cm rests on two parallel rails that are d = 12.0 cm apart and L = 45.0 cm long. The rod carries a current of I = 48.0 A (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field of magnitude 0.240 T is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails?

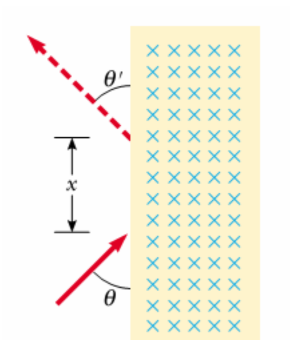


6. A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis, as shown in Figure. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.

7. A proton moving in the plane of the page has a kinetic energy of 6.00 MeV. A magnetic field of

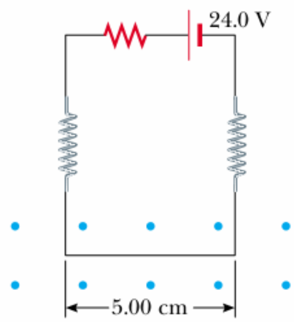


magnitude B = 1.00 T is directed into the page. The proton enters the magnetic field with its velocity vector at an angle $\theta = 45.0°$ to the linear boundary of the field as shown in Figure. (a) Find x, the distance from the point of entry to where the proton will leave the field. (b) Determine θ', the angle between the boundary and the proton's velocity vector as it leaves the field.

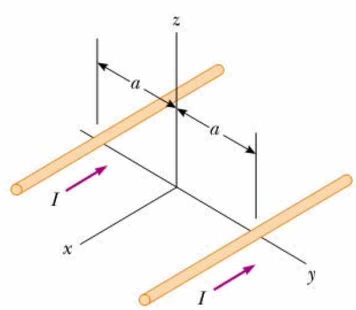


8. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^{6} m/s. (a) What is the magnitude of the magnetic moment due to the electron's motion? (b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of this magnetic moment vector?

9. The circuit in Figure consists of wires at the top and bottom and identical metal springs in the left and right sides. The upper portion of the circuit is fixed. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The springs stretch 0.500 cm under the weight of the wire and the circuit has a total resistance of 12.0 Ω. When a magnetic field is turned on, directed out of the page, the springs stretch an additional 0.300 cm. What is the magnitude of the magnetic field?



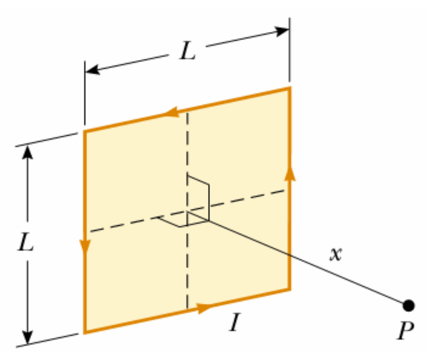
10. In Figure, both currents in the in finitely long wires are in the negative x direction. (a) Sketch the magnetic field pattern in the yz plane. (b) At what distance d along the z axis is the magnetic field a maximum?



11. What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

12. A wire is formed into the shape of a square of edge length L. Show that when the current in the loop is I, the magnetic field at point P, a distance x from the center of the square along its axis is

$$B = \frac{\mu_0 I L^2}{2\pi(x^2 + L^2/4)\sqrt{x^2 + L^2/2}}$$

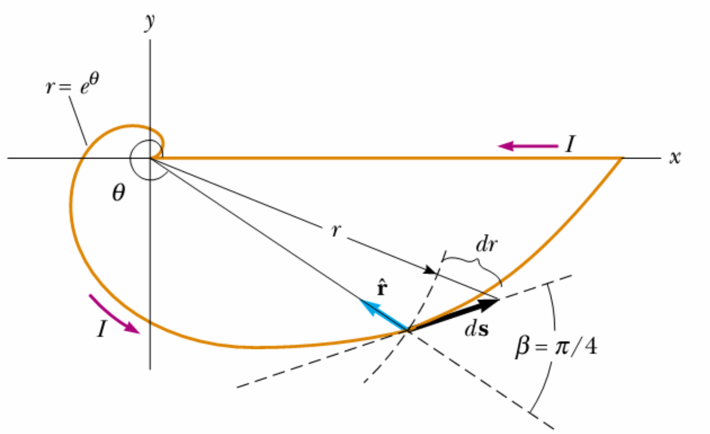


13. A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$, from $\theta = 0$ to $\theta = 2\pi$ as suggested in Figure. To complete a loop, the ends of the spiral are connected by a straight wire along the x axis. Find the magnitude and direction of **B** at the origin.
Suggestions: Use the Biot–Savart law. The angle β between a radial line and its tangent line at any point on the curve $r = f(\theta)$ is related to the function in the following way:

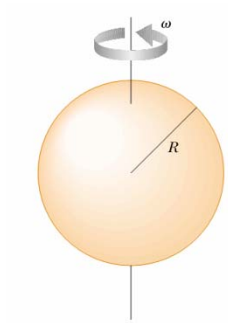
$$\tan\beta = \frac{r}{dr/d\theta}$$

Thus in this case $r = e^{\theta}$, $\tan\beta = 1$ and $\beta = \pi/4$. Therefore, the angle between **ds** and **r** is $\pi - \beta = 3\pi/4$. Also

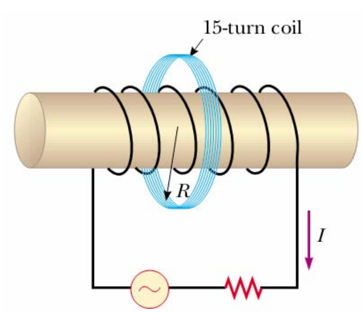
$$ds = \frac{dr}{\sin(\pi/4)} = \sqrt{2}\,dr$$



14. A sphere of radius R has a uniform volume charge density ρ. Determine the magnetic field at the center of the sphere when it rotates as a rigid object with angular speed ω about an axis through its center.



15. A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.00 cm and 1.00×10^{3} turns/meter. The current in the solenoid changes as I = (5.00 A) sin (120t). Find the induced emf in the 15-turn coil as a function of time.

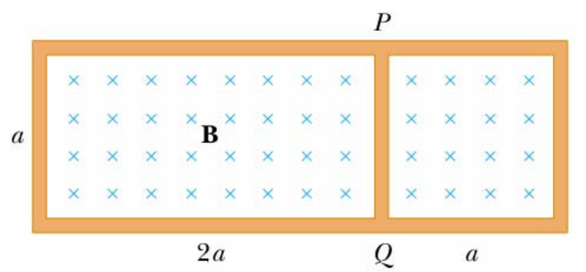


16. A rectangular loop of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = B_{max} e^{-t/\tau}$, where B_{max} and τ are constants. The field has the constant value B_{max} for $t < 0$.
(a) Use Faraday's law to show that the emf induced in the loop is given by

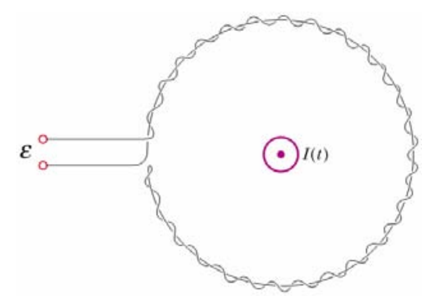
$$\boldsymbol{\varepsilon} = \frac{AB_{\max}}{\tau} e^{-t/\tau}$$

(b) Obtain a numerical value for $\boldsymbol{\varepsilon}$ at t = 4.00 s when A = 0.160 m^2, B_{max} = 0.350 T, and τ = 2.00 s. (c) For the values of A, B_{max}, and τ given in (b), what is the maximum value of $\boldsymbol{\varepsilon}$?

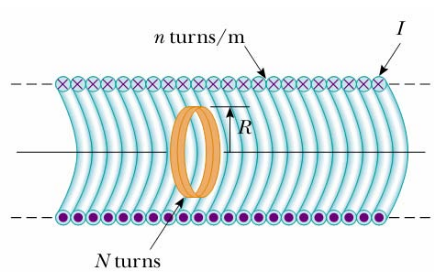
17. Find the current through section PQ of length a = 65.0 cm in Figure. The circuit is located in a magnetic field whose magnitude varies with time according to the expression B = $(1.00 \times 10^{-3}$ T/s)t. Assume the resistance per length of the wire is 0.100 Ω/m.



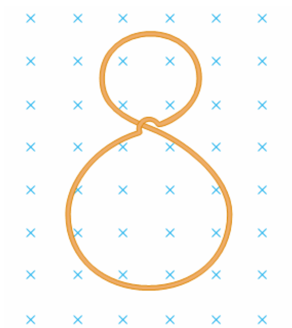
18. When a wire carries an AC current with a known frequency, you can use a Rogowski coil to determine the amplitude I_{max} of the current without disconnecting the wire to shunt the current in a meter. The Rogowski coil, shown in Figure, simply clips around the wire. It consists of a toroidal conductor wrapped around a circular return cord. The toroid has n turns per unit length and a cross-sectional area A. The current to be measured is given by $I(t) = I_{max} \sin \omega t$. (a) Show that the amplitude of the emf induced in the Rogowski coil is $\varepsilon_{max} = \mu_0 n A \omega I_{max}$. (b) Explain why the wire carrying the unknown current need not be at the center of the Rogowski coil, and why the coil will not respond to nearby currents that it does not enclose.



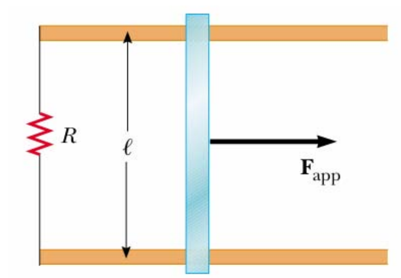
19. A long solenoid has n = 400 turns per meter and carries a current given by $I = (30.0 \text{ A})(1 - e^{-1.60 t})$. Inside the solenoid and coaxial with it is a coil that has a radius of 6.00 cm and consists of a total of N = 250 turns of fine wire. What emf is induced in the coil by the changing current?



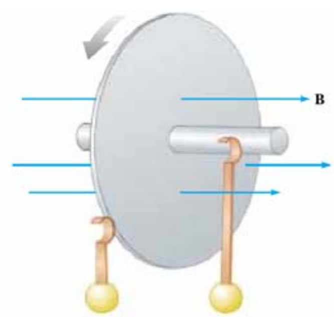
20. A piece of insulated wire is shaped into a figure 8, as in Figure. The radius of the upper circle is 5.00 cm and that of the lower circle is 9.00 cm. The wire has a uniform resistance per unit length of 3.00 Ω/m. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown. The magnetic field is increasing at a constant rate of 2.00 T/s. Find the magnitude and direction of the induced current in the wire.



21. Figure shows a top view of a bar that can slide without friction. The resistor is 6.00 Ω and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let l = 1.20 m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is energy delivered to the resistor?



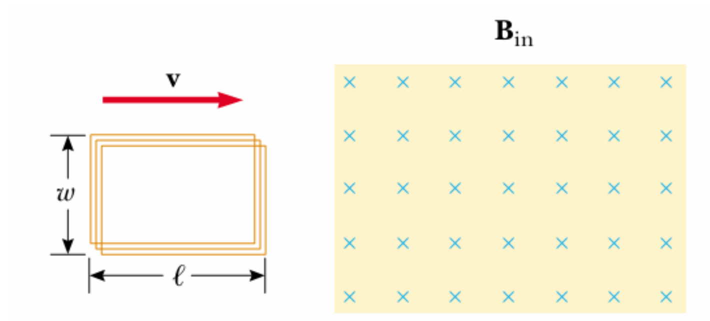
22. The homopolar generator, also called the Faraday disk, is a low voltage, high-current electric generator. It consists of a rotating conducting disk with one stationary y brush (a sliding electrical contact) at its axle and another at a point on its circumference, as shown in Figure. A magnetic field is applied perpendicular to the plane of the disk. Assume the field is 0.900 T, the angular speed is 3200 rev/min, and the radius of the disk is 0.400 m. Find the emf generated between the brushes. When superconducting coils are used to produce a large magnetic field, a homopolar generator can have a power output of several megawatts. Such a generator is useful, for example, in purifying metals by electrolysis. If a voltage is applied to the output terminals of the generator, it runs in reverse as a homopolar motor capable of providing great torque, useful in ship propulsion.



23. An automobile has a vertical radio antenna 1.20 m long. The automobile travels at 65.0 km/h on a horizontal road where the Earth's magnetic field is 50.0 /T directed toward the north and downward at an angle of 65.0° below the horizontal. (a) Specify the direction that the automobile should move in

order to generate the maximum motional emf in the antenna, with the top of the antenna positive relative to the bottom. (b) Calculate the magnitude of this induced emf.

24. A rectangular coil with resistance R has N turns, each of length l and width w as shown in Figure. The coil moves into a uniform magnetic field B with constant velocity v. What are the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field, (b) as it moves within the field, and (c) as it leaves the field?

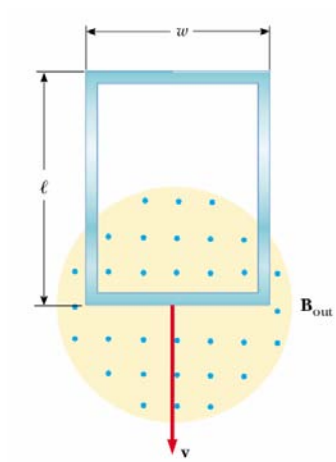


25. A long solenoid with 1000 turns per meter and radius 2.00 cm carries an oscillating current given by I = (5.00 A) sin(100πt). What is the electric field induced at a radius r = 1.00 cm from the axis of the solenoid? What is the direction of this electric field when the current is increasing counterclockwise in the coil?

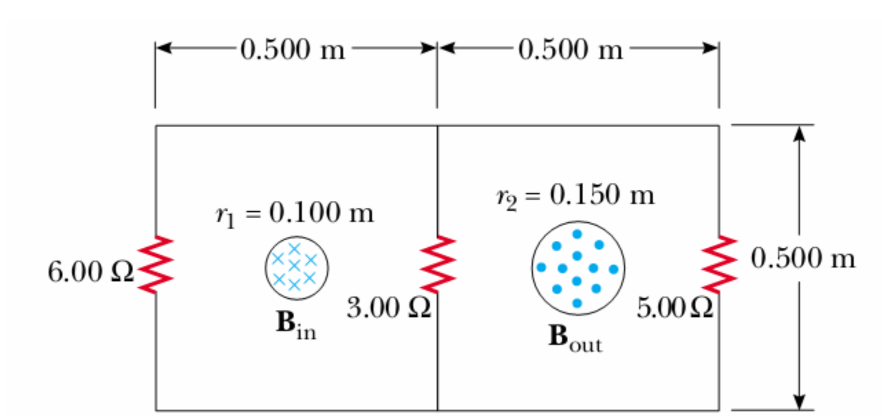
26. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field **B** as shown in Figure. During the time interval before the top edge of the loop reaches the field, the loop approaches a terminal speed v_T. (a) Show that

$$v_T = \frac{MgR}{B^2w^2}$$

(b) Why is v_T proportional to R? (c) Why is it inversely proportional to B^2?

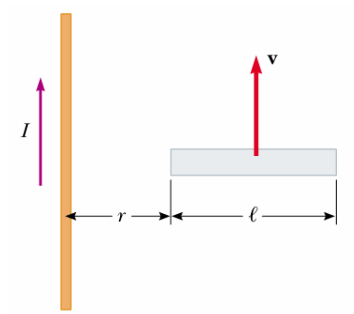


27. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Figure. The magnitude of **B** inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?



28. A conducting rod of length *l* moves with velocity v parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away, as shown in Figure. Show that the magnitude of the emf induced in the rod is

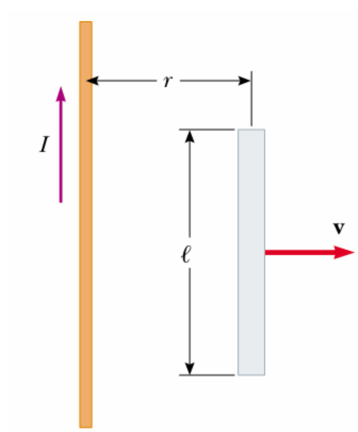
$$|\mathcal{E}| = \frac{\mu_0 I v}{2\pi} \ln\left(1 + \frac{\ell}{r}\right)$$



29. A conducting rod moves with a constant velocity **v** in a direction perpendicular to a long, straight wire carrying a current I as shown in Figure. Show that the magnitude of the emf generated between the ends of the rod is

$$|\mathcal{E}| = \frac{\mu_0 v I \ell}{2\pi r}$$

In this case, note that the emf decreases with increasing r, as you might expect.



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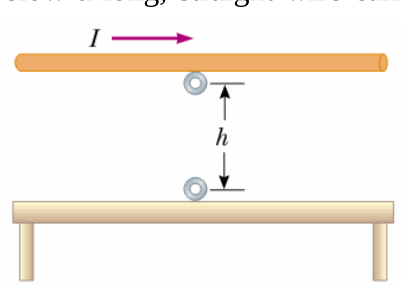
32. A small circular washer of radius 0.500 cm is held directly below a long, straight wire carrying a current of 10.0 A. The washer is located 0.500 m above the top of a table. (a) If the washer is dropped from rest, what is the



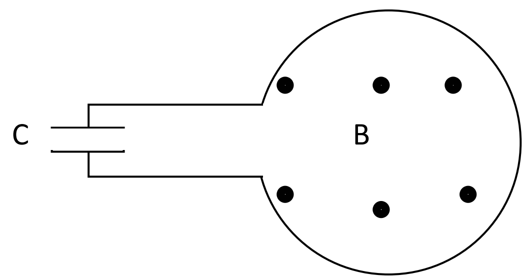
magnitude of the average induced emf in the washer from the time it is released to the moment it hits the tabletop? Assume that the magnetic field is nearly constant over the area of the washer, and equal to the magnetic field at the center of the washer. (b) What is the direction of the induced current in the washer?

Assignment 10: PHY 121

1. A solenoid of length l has a total of N number of turns with a cross sectional area A. Assuming the solenoid is very long apply Amperes law to derive an expression for the magnetic field in the solenoid. Calculate the total magnetic flux linked with the solenoid and from that the self inductance of the solenoid. Show that when a constant current I is passed through the solenoid the Energy stored in the solenoid is proportional to N^2.

2. A capacitor of capacitance C is charged to V_o volts by a battery and then disconnected from the battery. Now a resistor R is connected across the ends of the capacitor which slowly discharges. Write an differential equation to determine the amount of charge on the capacitor at any instant. Solve the equation to show that $Q=Q_o e^{-t/\tau}$, what are the values of Q_o and τ in terms of C, V_o and R.

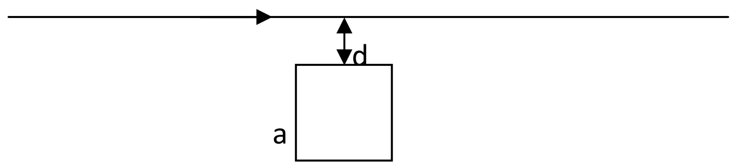
3. A circular ring of area 100 cm^2 is connected to a 15μF capacitor as shown below. The circuit has a resistance of 2Ω. A uniform but time dependent magnetic field of magnitude B = 0.3(T/s)t lies perpendicular to the plane of the loop. Calculate the induced emf, current in the ring and charge in the capacitor. Give the direction of current and polarity of charge. [Ans: constant induced emf = 3×10^{-4}V, $Q=4.5\times10^{-9}(1-e^{-t/\tau})$ where $\tau=3\times10^{-5}$s]



4. An electron is injected at t=0s with velocity $v_o=2\times10^6$m/s **i** into a region with parallel electric and magnetic field E = 1500 (V/m) **j** and B = -0.2 (T) **j**, respectively. Calculate the subsequent motion. [ans: the circular motion is parallel to the xz plane with radius 5.7×10^{-5}m. along y axis the particle has constant acceleration.]

5. (i) A wire loop has an self inductance 2 mH. Calculate the magnetic flux through the loop when a current of 30 mA passes through the loop.
(ii) The emf induced in an isolated circuit when the current changes by 10A/s is 0.3 V. Calculate the self inductance of the circuit.

6. the figure below shows a straight wire and a square loop (of side a) with one side oriented parallel to the wire, at a distance d. Calculate mutual inductance of the system. What is the direction of flow of the induced current in the square loop if current I flows in a direction given by the arrow along the wire and increases as a function of time.



7. An engineer constructs a cylindrical solenoid of area 8 cm^2 and length 25 cm from 150 cm of wire. The wire can handle a max current of 50 mA. What is the self inductance of the coil and what is the maximum energy that the solenoid can store?

8. Consider 2 inductors of self inductances L1 and L2 are connected in parallel in a circuit. Neglecting the Mutual inductance between the inductors what is the value of the equivalent inductance of this combination.

9. A long solenoid of radius R and n turns per unit length carriers a current $I = I_o \sin(\omega t)$. What is the electric field at distance R/2 (inside the solenoid) and distance 2R (outside the solenoid).?

10. A blood flow meter produces a 0.03T magnetic field perpendicular to the direction of blood flow. What is the magnetic force on an ion of charge e flowing in the blood at 0.25 m/s? +ve and -ve ions in the blood are forced in the opposite direction producing an electric field and a voltage across the blood vessel. the ion separation stops when the electric force caused by this voltage balances the magnetic force. What voltage will be developed across a 3 mm blood vessel?

11. Two parallel wires (of radius a) are a distance d apart and carry equal current in opposite directions. Show that the inductance per unit length of this pair of wires is given by $(\mu_o/\pi)\ln\{(d-a)/a\}$. Note: neglect the flux within the wires themselves.