Exam/PHY 121/2011-12/1M

Indian Institute of Science Education and Research Thiruvananthapuram
PHY 121 Electromagnetic Theory, First Mid-Semester Examination, Semester 2 (Vasanth), Batch 2011
4 February 2012, Duration: 60 minutes, Marks: 40

($1/4\pi\varepsilon_o = 9 \times 10^9$ Nm^2/C^2, g = 10 m/s^2)

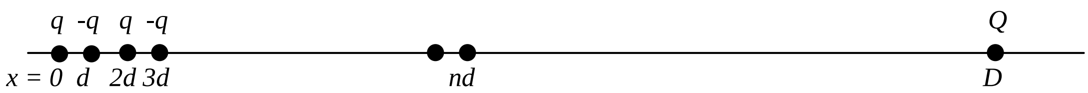
1) If the operator ∇^2 is defined as $\nabla.(\nabla\ \ )$ find $\nabla^2(\ln r)$. [4]

2) Check the validity of the divergence theorem for the vector field $r^5\mathbf{r}$ over a spherical surface of radius R_o centred at the origin and the enclosed volume. [6]

3) A vector field $\mathbf{a} = (z^2 + 2xy)\ \mathbf{i} + (x^2 + 2yz)\ \mathbf{j} + (y^2 + 2zx)\ \mathbf{k}$. Show that **a** is conservative and calculate the corresponding scalar potential. Calculate the line integral $\int \mathbf{a}\cdot d\mathbf{r}$ along any line joining the points (1, 1, 1) and (1, 2, 2). [6]

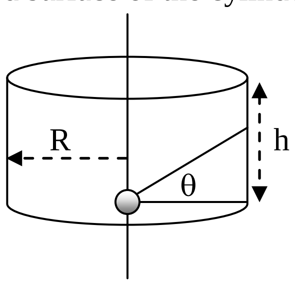
4) An infinite vertical sheet carries a charge density of 1.2×10^{-6} C/m^2. A ball of mass 8g is suspended by a string 50 cm long from a point, 55 cm from the charged sheet. Assuming gravity acts vertically downwards, calculate the angle the string makes with the vertical if a charge of (i) 0.8×10^{-8} C and (ii) $- 3.0 \times 10^{-8}$ C is placed on the ball. [6]

5) (n+1) alternating positive and negative point charges (q and -q) are placed at x = 0, x = d, x = 2d, x = 3d, ..., x = nd. An isolated charge Q is kept at x = D, (as shown in the figure below) D > nd. (i) Write a general expression for the total Force on the charge Q. (ii) Approximate and simplify your result for the limit D >> d.
(Note: (a) Assume n is an odd number (b)Use $(1+x)^{-2} \cong (1-2x)$ for x << 1 for answering part (ii)) [6]



6) A certain experiment requires an electric field that points symmetrically away from an axis and has a constant magnitude. Determine the charge distribution capable of generating such a field. [6]

7) A point charge Q is placed at the bottom centre of a cylinder of radius R and height h. Find the electric flux through the curved surface of the cylinder, by direct integration. (Use (as your integration variable) [6]

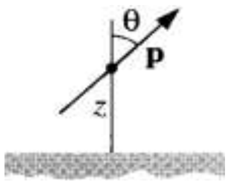


Indian Institute of Science Education and Research Thiruvananthapuram
PHY 121 Electromagnetic Theory, Second Mid-Semester Examination, Semester 2 (Vasanth), Batch 2011
12 March 2011, Duration: 60 minutes, Marks: 40

($1/4\pi\varepsilon_o = 9\text{x}10^9$ Nm^2/C^2, Q = CV, C = $\varepsilon_o A/d$, W = ½∫**D.E** dτ, **D** = ε_o**E**+**P** = ε**E**, $\varepsilon = \varepsilon_o(1+\chi) = \varepsilon_r\varepsilon_o$, **P**.n = σ_b, ∇.**P** = -ρ_b, W = -**p.E**, **N** = **p** x **E**)

1) The potential corresponding to a spherically symmetric charge distribution (of radius R) is given by;
(i) $V(r) = Q\,[-3 + 4(r/R)^2]\,/(4\pi\varepsilon_o R)$ for r < R
(ii) $V(r) = Q/4\pi\varepsilon_o r$ for r > R
Calculate the electric field in the two regions and the corresponding charge distributions. Plot V(**r**), **E**(**r**) and ρ(**r**) as a function of r. [10]

2) An electric dipole of moment **p** is situated at a distance *z* above an infinite grounded conducting plane, making an angle θ with the vertical, as shown in the figure. Find the torque on **p**. If the dipole is free to rotate (with z fixed) in which direction will it come to rest?[10]



3) A molecule of LiF, which has a permanent dipole moment, is placed in a uniform Electric field E = 10^4 N/C. If the difference between the maximum and the minimum potential energies of the dipole in this field is 4.4 x 10^{-25} J, calculate the electric dipole moment of the molecule. [5]

4) A parallel plate capacitor with a dielectric X between the plates is charged to 600 V. With the electric charge on the plates held fixed, the dielectric X is replaced with dielectric Y. What will be the new potential difference between the plates? Given the dielectric constant of X and Y are 2.5 and 3.5 respectively. [5]

5) Answer the following **briefly**:
(i) An uncharged metal plate is inserted midway between a parallel plate capacitor carrying charges Q and -Q on the plates. What will happen to the total energy of the system? [3]
(ii) The plates of an isolated parallel plate capacitor (carrying charge density +σ and -σ) are pulled slightly apart. What happens to the total energy of the system and its energy density? [3]
(iii) Draw the electric field lines for a point charge +q held at a distance *d* above a *thin* isolated conducting metal plate of area d^2, carrying a charge +q. How will field pattern change if the plane is now grounded? [4]

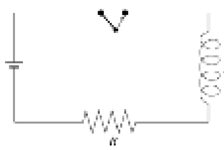
Indian Institute of Science Education and Research Thiruvananthapuram
PHY 121 Electromagnetic Theory, Final Examination, Semester 2 (Vasanth), Batch 2011
26 April 2012, Duration: 180 minutes, **Total Marks: 100**

Answer question 1, **any 5** questions from **Section A** and **any 3** questions from **Section B**. All plots/diagrams must be labelled completely to get credit.

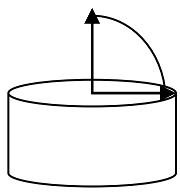
1) Answer the following **briefly**: **[29 marks]**
(i) A charge Q is uniformly distributed over the surface of a spherical balloon. How does the electric field vary for a point inside, on the surface and outside the balloon, as it is blown up?**[4]**
(ii) In electrostatics if the Electric field at a certain point is zero, the potential at that point must also be zero. Is the statement true or false? Discuss with 2 examples **[4]**
(iii) Imagine a physical system in which the dielectric constant is less than 1. How should such a system respond when placed in an external electric field? **[3]**
(iv) You are given a sheet of metal of area A. You can transform it into (i) a single spherical shell of radius R or (ii) 10 spherical shells of radius r. Assuming that in both cases exactly the whole sheet is used up, which of the two configurations will have a higher net capacitance? **[4]**
(v) Will a conductor always be an equipotential? If not under what circumstances will it not? **[3]**
(vi) A ring of metal, placed between the pole pieces of a permanent magnet with its plane perpendicular to the field lines, is removed. Does the amount of Joule heating produced in the ring depend on the exact rate at which it was removed? What about the total charge flowing through the ring, does that depend on the rate of removal? **[4]**
(vii) A inductor (L) and a resistor (R) are connected in series across a DC battery of emf V_o. Can the induced back emf ever be larger than the battery emf? **[3]**
(viii) The magnetic field outside an infinite solenoid carrying a constant current is zero. If the current in the solenoid is time varying the associated magnetic field will also vary with time. What happens to the magnetic and the electric field outside the solenoid? **[4]**

SECTION A

(Answer any 5 questions; each question carries 7 marks **[5 x 7 = 35]***)*



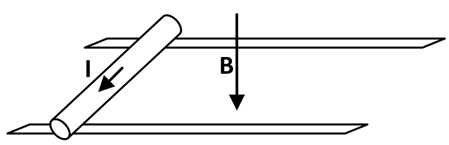
2) You wish to construct a circuit with an inductor L and a resistor connected in series with a DC battery, such that when the switch is closed the current builds up to 18% of its steady state value in 0.5 ms. If the value of the inductor chosen is 10 mH what should be the value of the resistor R?



3) The steady current density, pointed along the axis of a infinitely long cylindrical wire of radius R varies radially as $\mathbf{J} = J_o(1-r^2/R^2)\,\hat{z}$. Calculate the magnetic field **B** at a distance r lying (i) inside and (ii) outside the wire.

4) A point dipole **p** is located at the origin aligned vertically. Assuming the approximate electrostatic potential for a large distance *r*, compute the components of the electric field in the spherical polar coordinate system. Calculate the flux of the electric field over a surface of a sphere of radius r centred at the origin. Does this approximate field and the charge distribution obey Gauss' Law?

5) The uniform magnetic field perpendicular to the plane of a metal ring (of diameter 5 cm) increases at a linear rate from 0 to 2.5 T in 50 ms. If the resistance of ring is 0.2 Ω, calculate the total charge that flows through the ring in the time interval of 50 ms.

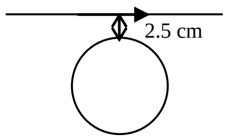


6) A rod of mass 500 g and radius 5.00 cm rests on two parallel rails that are d = 10.0 cm apart and L = 50 cm long. The rod carries a constant current of I = 50.0 A (in the direction shown) and can roll along the rails without slipping. A uniform magnetic field of magnitude 0.240 T (in the direction shown) is directed perpendicular to the rod and the rails. If it starts from rest, at the left extreme of the rails, calculate the speed of the rod as it leaves the rails?

7) A particle of charge +q and mass m moves at a constant speed $5.6\text{x}10^7$m/s along the +*x* direction into a region of uniform magnetic field B = 2.5 T, oriented along +*y* direction. Draw a schematic depicting the motion of the particle in the magnetic field and the direction of the force on the particle. If the radius of the circle traced is R = 250 cm what is the ratio q/m for the charged particle?

8) The *xy* plane contains a uniformly distributed, infinite sheet of point dipoles with n dipoles per unit area, each dipole having a moment p. Assuming, all the dipoles are oriented along the z axis, calculate the potential and electric field anywhere in the positive z space.

9) A Hall probe operates at 150-mA current. When placed perpendicular to a uniform magnetic field of magnitude 100 mT, it produces a Hall voltage of 500 μV. When placed perpendicular to an unknown magnetic field, the Hall voltage is 330 μV. Calculate the magnitude of the unknown field?



10) A circular loop of wire, of radius R (= 5 cm) lays 2.5 cm beside an infinitely long wire carrying a current I (= 2 A) to the right. What must be the direction and magnitude of the current in the circular loop such that the magnetic field at the centre of the loop is zero?

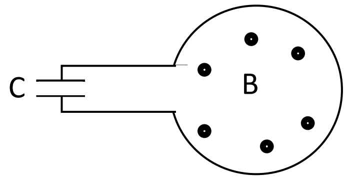
SECTION B

(Answer any 3 questions; each question carries 10 marks **[3 x 12 = 36]***)*

11) A spherically symmetric potential is given by V(r) = $\dfrac{\alpha}{4\pi\varepsilon_o}\dfrac{e^{-r/r_o}}{r}$; over all space. Given that α and r_o are positive constants, derive an expression for the electric field and the corresponding charge distribution. Plot $\rho(r)$ as a function of r and calculate the total charge by evaluating the integral $\int\rho(r)\, d\tau$.

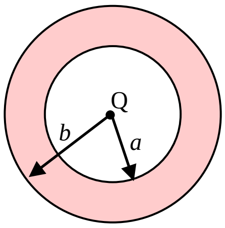
12) Derive an expression for the magnetic field at the centre of a thin square made of n loops of wire, of side 2a and carrying a current I in the clockwise direction, when looked down from above. Calculate the magnitude of the field if n = 100, a = 5 cm and I = 10A.

13) A circular ring of area 100 cm^2 and resistance 2Ω is connected to a C = 15μF capacitor as shown. At time t = 0, a spatially uniform magnetic field with its magnitude increasing linearly with time, given by B = 0.5(T/s)t (pointing out perpendicular to the plane of the ring) is switched on.



(i) Calculate the induced emf
(ii)Write the differential equation for the charge q on the capacitor.
(iii) Solve the above equation to obtain the time dependence of the charge on the capacitor and current in the circuit.
(ii) Plot the induced emf and the charge on the capacitor as a function of time.

14) A point charge Q is enclosed by a thick spherical shell, of inner radius *a* and outer radius *b*, made of a dielectric material which is <u>permanently polarized</u> with **P**(**r**) = k**r**/r^2, calculate the electric field and the electric displacement in the three regions of space r ≤ *a*, *a* < r ≤ *b* and r > *b*. Calculate all the bound volume and surface charge densities.



15) The figure below shows a infinite straight wire, carrying a constant current I and a square loop (of side a) with one side oriented parallel to the wire, at a distance d. At time t = 0 the loop is moved with a constant velocity $\mathbf{v_o}$ perpendicularly away from the infinite wire. Calculate (i) the flux enclosed by the loop at any instant (ii) emf induced in the loop as a function of time. Plot induced emf in the square loop as a function of time.